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### Semester Two Examination, 2019

### Question/Answer booklet

# MATHEMATICS SPECIALIST

**UNIT 2**

## Section One:

## Calculator-free

Your Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your Teacher’s Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Question | Mark | Max | Question | Mark | Max |
| 1 |  | 3 | 6 |  | 8 |
| 2 |  | 6 | 7 |  | 6 |
| 3 |  | 7 | 8 |  | 4 |
| 4 |  | 8 | 9 |  | 3 |
| 5 |  | 7 |  |  |  |

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 9 | 9 | 50 | 52 | 36 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 94 | 64 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.**Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**See Next Page**

**Section One: Calculator-free (52 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

● Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

● Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

**Question 1 (3 marks)**



Prove the following statement:

If and are each 1 less than a multiple of 3, then is 1 more than a multiple of 3.

|  |
| --- |
| **Solution** |
| Assume that and are each 1 less than a multiple of 3.  Then and for some .  Hence  which is 1 more than a multiple of 3 since is an integer.  QED |
| **Specific behaviours** |
| ✓ writes and as and  ✓ multiplies and simplifies to  ✓ concludes that is 1 more than a multiple of |

**Question 2 (6 marks)**



Consider the system of simultaneous linear equations:

1. Write down the matrix such that the equation

is equivalent to the system of equations above. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct matrix |

1. Suppose that is singular (non-invertible).
2. Determine the value of (show working). (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ attempts to solve for  ✓ states correct value |

1. State the possible number(s) of solutions that the system of equations could have with the value of you just found. (2 marks)

|  |
| --- |
| **Solution** |
| Either no solutions or infinitely many solutions. |
| **Specific behaviours** |
| ✓ states at least one of ‘no solutions’ and ‘infinitely many solutions’  ✓ states both possibilities  No marks if implies that there can be exactly one solution |

1. State the number of solutions the system has if has the value found above and . (1 mark)

|  |
| --- |
| **Solution** |
| No solutions. |
| **Specific behaviours** |
| ✓ States no solutions. |

**Question 3 (7 marks)**



1. Write in the form . (3 marks)

|  |
| --- |
| **Solution** |
| So |
| **Specific behaviours** |
| ✓ writes correct value for  ✓ writes correct value for  ✓ writes correct value for |

1. Hence, solve the equation for . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ equates expression from part (a) to  ✓ obtains at least one general solution (using or ) for  ✓ obtains at least 3 correct solutions for (in correct domain)  ✓ obtains 6 correct solutions for (in correct domain) |

**Question 4 (8 marks)**



A real matrix can ‘transform’ a complex number if we view the complex number as a column vector. That is, for any complex number , the matrix transforms to where .

Find the matrix which (according to this rule) will transform any complex number to:

1. (2 marks)

|  |
| --- |
| **Solution** |
| Multiplication by 3 corresponds to dilation by factor 3. |
| **Specific behaviours** |
| ✓ identifies corresponding transformation  ✓ writes correct matrix |

1. (2 marks)

|  |
| --- |
| **Solution** |
| Conjugation corresponds to reflection in -axis. |
| **Specific behaviours** |
| ✓ identifies corresponding transformation  ✓ writes correct matrix |

1. (2 marks)

|  |
| --- |
| **Solution** |
| Multiplying by corresponds to rotating anti-clockwise by . |
| **Specific behaviours** |
| ✓ identifies corresponding transformation  ✓ writes correct matrix |

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes product of matrices OR identifies corresponding transformation sequence  ✓ writes correct matrix |

Question 5 (7 marks)



Evaluate the following for complex numbers and

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct calculation  ✓ writes correct answer |

1. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes correct expression for conjugate of  ✓ writes correct answer |

1. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ multiplies numerator and denominator by  ✓ expands numerator and denominator correctly  ✓ writes correct answer |

**Question 6 (8 marks)**



In this question, a *proper* factor is a factor greater than 1.

Assume that and are both integers, and consider the following statement:

If has no proper square factors, then neither nor has a proper square factor.

1. Prove the statement using the method of proof by contradiction. (3 marks)

|  |
| --- |
| **Solution** |
| Assume that has no proper square factors, but that either or has a proper square factor.  Then either or for some .  Hence either or , and in each case has a proper square factor.  This is a contradiction, and so neither nor has a proper square factor. |
| **Specific behaviours** |
| ✓ assumes negation of statement  ✓ writes and/or as a product of an integer and a square  ✓ shows that must therefore have a square factor and notes contradiction  Accept argument for alone having a square factor together with recognition that a similar argument will apply if has a square factor. |

1. Write the converse of the statement. (2 marks)

|  |
| --- |
| **Solution** |
| If neither nor has a proper square factor, then has no square factors. |
| **Specific behaviours** |
| ✓ ✓ writes correct converse statement |

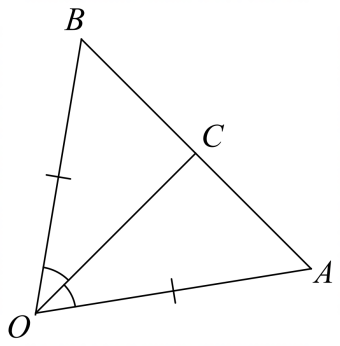
1. Write whether the converse is true or false and prove or disprove it accordingly.

(3 marks)

|  |
| --- |
| **Solution** |
| False. E.g. if and , then neither nor has a proper square factor, but . |
| **Specific behaviours** |
| ✓ states false  ✓✓ gives correct counterexample |

**Question 7 (6 marks)**



Let be the origin, let and be points such that , and let be a point on such that bisects .

Let , and .

a) Show that . (3 marks)

|  |
| --- |
| **Solution** |
| Let . Then .  Now and .  Since it follows that . |
| **Specific behaviours** |
| ✓ writes expressions for and  ✓ uses the fact that  ✓ uses the fact that |

b) **Hence**, prove that is perpendicular to . (3 marks)

|  |
| --- |
| **Solution** |
| (by part a)  Hence is perpendicular to . |
| **Specific behaviours** |
| ✓ writes as  ✓ attempts to determine dot product  ✓ shows that |

Question 8 (4 marks)



Prove the following identity.

|  |
| --- |
| **Solution** |
| Hence LHS = RHS QED |
| **Specific behaviours** |
| ✓ rearranges RHS (or LHS) to obtain LHS (or RHS)  ✓ correctly uses sum-to-product identities  ✓ cancels factor of  ✓ simplifies to RHS (or LHS) |

**Question 9 (3 marks)**



Let be a line containing a point , and let be a point not on . Suppose that is a unit vector perpendicular to the line . Prove that the perpendicular distance from to is .

|  |
| --- |
| **Solution** |
| Distance from to is  Hence (or if has opposite direction). |
| **Specific behaviours** |
| ✓ writes as  ✓ uses the fact that  ✓ uses the fact that the shortest distance from to is  (Note that only is required for marks.) |

**END OF SECTION ONE**

Additional working space

Question number:

Additional working space

Question number: